بسم الله الرحمن الرحيم

King Abdulaziz University

Review for The First Exam-Fall 2012

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على ستة وثلاثون سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus II
Math202

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Version 1.0

Enter Name:

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Start Answer each of the following.

1. $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) =$

- 0
- $2x$
- e
- 1

2. $\operatorname{sech} 0 =$

- ☐ 0
- ☐ e
- ☐ 1
- ☐ undefined

3. If $y = \operatorname{sech}^3\left(x^2 + e^x\right)$, then

- ☐ $y' = 3(2x + e^x)\operatorname{sech}^3\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$
- ☐ $y' = -3(2x + e^x)\operatorname{sech}^3\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$
- ☐ $y' = (2x + e^x)\operatorname{sech}^3\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$
- ☐ $y' = 3(2x + e^x)\operatorname{sech}^2\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$

4. $1 - \tanh^2 x =$

- ☐ 1
- ☐ -1
- ☐ $-\operatorname{sech}^2 x$
- ☐ $\operatorname{sech}^2 x$

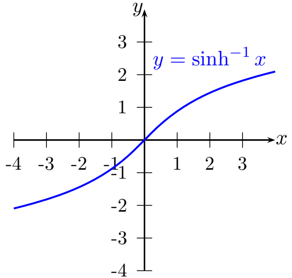
5. If $0 < x < 1$ and $y = \operatorname{sech}^{-1}(\sqrt{1-x^2})$, then $y' =$

- ☐ $\frac{1}{1-x^2}$
- ☐ $\frac{-1}{1-x^2}$
- ☐ $\frac{1}{\sqrt{1-x^2}}$
- ☐ $\frac{-1}{\sqrt{1-x^2}}$

6. If F is an antiderivative of f on an interval I, then $G(x) = F(x) + e^2$ is also an antiderivative of f.

- ☐ TRUE
- ☐ FALSE

7. $\lim_{x\to\infty} \sinh^{-1} x =$



- ☐ ∞
- ☐ $-\infty$
- ☐ 1
- ☐ -1

8. If $f'(x) = 5e^x + 7/(1+x^2)$ and $f(0) = 3$, then $f(x) =$

- ☐ $5e^x + 7\tan^{-1}x - 2$
- ☐ $5e^x + 7\tan^{-1}x + 2$
- ☐ $5e^x + 7\cot^{-1}x - 2$
- ☐ $5e^x + 7\cot^{-1}x + 2$

9. If $f'(x) = 5e^x + 7/(1+x^2)$ and $f(0) = 3$, then $f(x) =$

- ☐ $5e^x + 7\tan^{-1}x - 2$
- ☐ $5e^x + 7\tan^{-1}x + 2$
- ☐ $5e^x + 7\cot^{-1}x - 2$
- ☐ $5e^x + 7\cot^{-1}x + 2$

10. The sigma notation of $2^3 + 3^3 + 4^3 + \ldots + n^3$ is

- ☐ $\sum_{i=-2}^{n-4} (i+4)^3$

- ☐ $\sum_{i=-1}^{n-4} (i+4)^3$

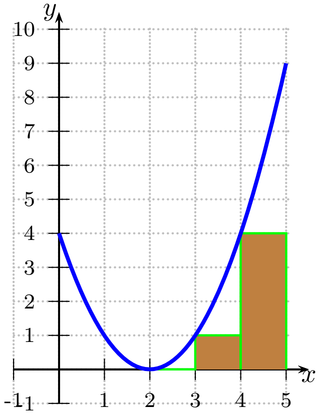
- ☐ $\sum_{i=0}^{n-4} (i+4)^3$

- ☐ $\sum_{i=1}^{n-4} (i+4)^3$

11. $\sum_{i=1}^{11}(i+2)$ is

- ☐ 88
- ☐ 86
- ☐ 77
- ☐ 76

12. A lower estimate of the area under the curve of $f(x) = x^2 - 4x + 4$ and the x-axis from $x = 2$ to $x = 5$ using three rectangles is



- ☐ 5
- ☐ 14
- ☐ 3
- ☐ 11

13. The integral expression of $\lim_{n\to\infty}\sum_{i=1}^{n}(2x_i^3 + x_i\cos x_i)\triangle_x$ over the interval $[0,\pi]$ is

☐ $\int_0^\pi (2x + x\cos x)\,dx$

☐ $\int_0^\pi (x^3 + x\cos x)\,dx$

☐ $\int_0^\pi (x + x\cos x)\,dx$

☐ $\int_0^\pi (2x^3 + x\cos x)\,dx$

14. If $\int_1^2 f(x)\,dx = 5$, then $\int_1^2 4f(u)\,du =$

- ☐ 20
- ☐ 5
- ☐ $\frac{5}{4}$
- ☐ 9

15. If $\int_{1}^{10} f(x)dx = 12$ and $\int_{7}^{10} f(x)dx = 10$, then $\int_{1}^{7} 5f(x)dx =$

- ☐ 2
- ☐ 4
- ☐ 10
- ☐ 6

16. $\frac{d}{dx}\left(\int_{1}^{x^4} \cosh t\, dt\right) =$

- ☐ $\cosh\left(x^4\right)$
- ☐ $\sinh\left(x^4\right)$
- ☐ $4x^3 \cosh\left(x^4\right)$
- ☐ $4x^3 \sinh\left(x^4\right)$

17. $\int\limits_{1}^{9} \frac{x-1}{\sqrt{x}}\,dx =$

- ☐ 10
- ☐ 14
- ☐ $\frac{-40}{3}$
- ☐ $\frac{40}{3}$

18. If $-2 \leq f(x) \leq 5$ for all $x \in [1, 4]$ then

- ☐ $-6 \leq \int_1^4 f(x)\,dx \leq 15$
- ☐ $-2 \leq \int_1^4 f(x)\,dx \leq 5$
- ☐ $-5 \leq \int_1^4 f(x)\,dx \leq 8$
- ☐ $-8 \leq \int_1^4 f(x)\,dx \leq 2$

19. $\int_{3}^{3} \frac{\sinh^{-1} x}{2x}\, dx =$

- ☐ e^2
- ☐ $-e^2$
- ☐ 0
- ☐ $\frac{1}{e^2}$

20. $\int_{-3}^{3} \frac{\sinh x}{x^4 + x^2}\, dx =$

- ☐ $\frac{e^2+2}{32}$
- ☐ 0
- ☐ $\frac{e^2-2}{32}$
- ☐ $\frac{e^2}{32}$

21. $\int \frac{\sin(\ln x)}{x}\, dx =$

- ☐ $\sin(\ln x) + C$
- ☐ $-\sin(\ln x) + C$
- ☐ $\cos(\ln x) + C$
- ☐ $-\cos(\ln x) + C$

22. $\int_{e}^{e^4} \frac{2}{x\sqrt{\ln x}}\, dx =$

- ☐ 4
- ☐ 8
- ☐ 6
- ☐ $e^4 - e$

23. $\int x \csc^2(x^2)\, dx =$

- ☐ $\cot(x^2) + C$
- ☐ $\frac{-\cot(x^2)}{2} + C$
- ☐ $-\cot(x^2) + C$
- ☐ $\frac{\cot(x^2)}{2} + C$

24. $\int_{\pi/6}^{\pi/3} \sec\theta \tan\theta \, d\theta =$

- ☐ 2
- ☐ 0
- ☐ $2 - \frac{2}{\sqrt{3}}$
- ☐ $\frac{1}{2} - \frac{\sqrt{3}}{2}$

25. If $\int f(x)\,dx = F(x) + C$ then $\frac{d}{dx}\left(F(x) + \operatorname{csch} x\right) =$

- ☐ $f(x) + \coth x$
- ☐ $f(x) + \operatorname{csch} x \coth x$
- ☐ $f'(x) - \coth x$
- ☐ $f(x) - \operatorname{csch} x \coth x$

26. If $\int \frac{20x+4}{\sqrt[3]{5x^2+2x+1}}\,dx =$

- ☐ $\sqrt{(5x^2+2x+1)^2} + C$
- ☐ $\sqrt[3]{(5x^2+2x+1)^2} + C$
- ☐ $3\sqrt[3]{(5x^2+2x+1)^2} + C$
- ☐ $3\sqrt{(5x^2+2x+1)^2} + C$

27. If $\int 5^{x^2+x}(6x+3)\,dx =$

- ☐ $\frac{5^{x^2+x}}{\ln 5} + C$
- ☐ $3\frac{5^{x^2+x}}{\ln 5} + C$
- ☐ $\frac{5^{x^2+x}}{3\ln 5} + C$
- ☐ $\frac{5^{x^2+x}\ln 5}{3} + C$

28. If $\int_0^2 |3-3x|\,dx =$

- ☐ 3
- ☐ -3
- ☐ 0
- ☐ 1

29. If g is an even function and $\int_{-3}^{3} g(x)\,dx = 14$, then $\int_{0}^{3} \frac{g(x)}{7}\,dx =$

- ☐ 14
- ☐ 2
- ☐ 7
- ☐ 1

30. $\int \frac{e^x}{1+e^x}\,dx =$

- ☐ $\ln(1+e^x) + C$
- ☐ $\frac{(1+e^x)^2}{2} + C$
- ☐ $\tan^{-1}(e^x) + C$
- ☐ $\sqrt{1+e^x} + C$

31. $\int \frac{e^x}{1+e^{2x}}\,dx =$

- ☐ $\ln\left(1+e^x\right)+C$
- ☐ $\frac{(1+e^x)^2}{2}+C$
- ☐ $\tan^{-1}\left(e^x\right)+C$
- ☐ $\sqrt{1+e^x}+C$

32. $\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx =$

- ☐ $2\sin\left(\sqrt{x}\right)+C$
- ☐ $\sin\left(\sqrt{x}\right)2+C$
- ☐ $\tan\left(\sqrt{x}\right)+C$
- ☐ $\sqrt{\sin x}+C$

33. $\int x\cos(5x)\,dx =$

- ☐ $\frac{x}{5}\sin(5x) - \frac{1}{25}\cos(5x) + C$
- ☐ $\frac{x}{5}\sin(5x) + \frac{1}{25}\cos(5x) + C$
- ☐ $\frac{x}{5}\sin(5x) + \frac{1}{5}\cos(5x) + C$
- ☐ $\frac{x}{5}\sin(5x) - \frac{1}{5}\cos(5x) + C$

34. $\int 3x^2e^{5x}\,dx =$

- ☐ $\frac{3}{5}x^2e^{5x} + \frac{6}{25}xe^{5x} - \frac{6}{125}e^{5x} + C$
- ☐ $\frac{3}{5}xe^{5x} - \frac{6}{25}e^{5x} + \frac{6}{125}x^2e^{5x} + C$
- ☐ $\frac{3}{5}x^2e^{5x} - \frac{6}{25}xe^{5x} + \frac{6}{125}e^{5x} + C$
- ☐ $\frac{3}{5}x^2e^{5x} - \frac{6}{25}xe^{5x} - \frac{6}{125}e^{5x} + C$

35. $\int e^x \cos x \, dx =$

- ☐ $\frac{e^x \sin x}{2} + C$
- ☐ $\frac{e^x (\cos x - \sin x)}{2} + C$
- ☐ $\frac{e^x (\sin x + \cos x)}{2} + C$
- ☐ $\frac{e^x (\sin x - \cos x)}{2} + C$

36. $\int \ln(2x - 1) \, dx =$

- ☐ $\frac{2x-1}{2} \ln(2x - 1) + x + C$
- ☐ $\frac{2x+1}{2} \ln(2x - 1) - x + C$
- ☐ $\frac{2x-1}{2} \ln(2x - 1) - x + C$
- ☐ $\frac{2x+1}{2} \ln(2x - 1) + x + C$

End

Score: Correct

Answers:

Points: [ ] Percent: [ ]

Letter Grade: [ ]

Solutions to Quizzes

Solution to 1.

$$\begin{aligned}\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) &= \ln(e^x) + \ln(e^{-x}) \\ &= x + (-x) \\ &= 0.\end{aligned}$$

Another solution:

$$\begin{aligned}\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) &= \\ &\ln((\cosh x + \sinh x)(\cosh x - \sinh x)) \\ &= \ln(\cosh^2 x - \sinh^2 x) \\ &= \ln 1 = 0.\end{aligned}$$

■

Solution to 2.

$$\begin{aligned}
\text{Since } \operatorname{sech} x &= \frac{2}{e^x + e^{-x}} \\
\text{then } \operatorname{sech} 0 &= \frac{2}{e^0 + e^{-0}} \\
&= \frac{2}{e^0 + e^0} \\
&= \frac{2}{1+1} \\
&= 1
\end{aligned}$$

■

Solution to 3.

$$
\begin{aligned}
y &= \left(\operatorname{sech}\left(x^2+e^x\right)\right)^3 \\
y' &= 3\left(\operatorname{sech}\left(x^2+e^x\right)\right)^2(-\operatorname{sech}\left(x^2+e^x\right)\tanh\left(x^2+e^x\right))(2x+e^x) \\
&= 3\operatorname{sech}^2\left(x^2+e^x\right)(-\operatorname{sech}\left(x^2+e^x\right)\tanh\left(x^2+e^x\right))(2x+e^x) \\
&= -3(2x+e^x)\operatorname{sech}^3\left(x^2+e^x\right)\tanh\left(x^2+e^x\right).
\end{aligned}
$$

■

Solution to 4. Remember that

$$\begin{aligned}\cosh^2 x - \sinh^2 x &= 1 && \text{divide by } \cosh^2 x \text{ all sides}\\ \frac{\cosh^2 x}{\cosh^2 x} - \frac{\sinh^2 x}{\cosh^2 x} &= \frac{1}{\cosh^2 x} && \text{simplify}\\ 1 - \tanh^2 x &= \operatorname{sech}^2 x.\end{aligned}$$

■

Solution to 5.

$$y = \operatorname{sech}^{-1}\left(\sqrt{1-x^2}\right)$$

Use the fact $\left(\operatorname{sech}^{-1}(f(x))\right)' = \dfrac{-f'(x)}{f(x)\sqrt{1-[f(x)]^2}}$

$$y' = \frac{-\frac{-2x}{2\sqrt{1-x^2}}}{\sqrt{1-x^2}\sqrt{1-(\sqrt{1-x^2})^2}}$$

$$= \frac{\not{2}x}{\not{2}\sqrt{1-x^2}\sqrt{1-x^2}\sqrt{1-(1-x^2)}}$$

$$= \frac{x}{(1-x^2)\sqrt{1-1+x^2}}$$

$$= \frac{x}{(1-x^2)\sqrt{x^2}}$$

use the fact $|x| = \sqrt{x^2}$ and $|x| = x$ if $x > 0$

$$= \frac{x}{(1-x^2)x} = \frac{1}{1-x^2}.$$

■

Solution to 6. Remember that a function F is antiderivative of f if $F'(x) = f(x)$ for all $x \in I$. Now $G(x) = F(x)+e^2$, then $G'(x) = F'(x) + (e^2)' = f(x) + 0 = f(x)$. Thus G is antiderivative of f. ■

Solution to 7. Using the graph we see as x getting bigger and bigger, $\sinh^{-1} x$ is getting bigger.
Hence $\lim_{x\to\infty} \sinh^{-1} x = \infty$ ■

Solution to 8.

$$
\begin{aligned}
f'(x) &= 5e^x + 7\frac{1}{1+x^2} \\
f(x) &= 5e^x + 7\tan^{-1} x + C \\
3 = f(0) &= 5e^0 + 7\tan^{-1} 0 + C \\
3 &= 5 + C \\
C &= -2 \\
f(x) &= 5e^x + 7\tan^{-1} x - 2
\end{aligned}
$$

■

Solution to 9.

$$\begin{aligned} f'(x) &= 5e^x + 7\frac{1}{1+x^2} \\ f(x) &= 5e^x + 7\tan^{-1} x + C \\ 3 = f(0) &= 5e^0 + 7\tan^{-1} 0 + C \\ 3 &= 5 + C \\ C &= -2 \\ f(x) &= 5e^x + 7\tan^{-1} x - 2 \end{aligned}$$

■

Solution to 10.

$$2^3 + 3^3 + 4^3 + \ldots + n^3 = \sum_{k=2}^{n} k^3$$

we want to start with -2. we have $k = 2 \Rightarrow k - 4 = -2$

let $i = k - 4$ then $k = i + 4$ when $k = 2 \Rightarrow i = -2$

when $k = n \Rightarrow i = n - 4$

$$\begin{aligned} 2^3 + 3^3 + 4^3 + \ldots + n^3 &= \sum_{k=2}^{n} k^3 \\ &= \sum_{i=-2}^{n-4} (i+4)^3. \end{aligned}$$

■

Solution to 11.

$$\begin{aligned}\sum_{i=1}^{11}(i+2) &= \sum_{i=1}^{11} i + \sum_{i=1}^{11} 2 \\ &= \sum_{i=1}^{11} i + 2\sum_{i=1}^{11} 1 \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{ and } \sum_{i=1}^{n} 1 = n \\ &= \frac{11(11+1)}{2} + 2(11). \\ &= 66 + 22 = 88.\end{aligned}$$

■

Solution to 12. Since $f(x) = x^2 - 4x + 4$ is increasing on $[2, 5]$, then we will have a lower estimate of the area if we use the left endpoints. Now, $\triangle x = \frac{5-2}{3} = \frac{3}{3} = 1$. Hence the intervals are $[2, 3]$, $[3, 4]$, $[4, 5]$. The lower estimate is

$$\begin{aligned} L_3 &= f(2)\triangle x + f(3)\triangle x + f(4)\triangle x \\ &= (2^2 - 4(2) + 4)(1) + (3^2 - 4(3) + 4)(1) + (4^2 - 4(4) + 4)(1) \\ &= (0) + (1) + (4) = 5. \end{aligned}$$

■

Solution to 13.

$$\lim_{n\to\infty}\sum_{i=1}^{n}(2x_i^3 + x_i\cos x_i)\triangle_x = \int\limits_0^{\pi}(2x^3 + x\cos x)\,dx$$

■

Solution to 14.

$$\int_{1}^{2} 4f(u)\,du = 4\int_{1}^{2} f(u)\,du$$
$$= 4(5) = 20$$

■

Solution to 15.

$$\begin{aligned}\int_1^7 5f(x)\,dx &= 5\int_1^7 f(x)\,dx \\ &= 5\left[\int_1^{10} f(x)\,dx - \int_7^{10} f(x)\,dx\right] \\ &= 5[12-10] = 10.\end{aligned}$$

■

Solution to 16.

$$\frac{d}{dx}\left(\int_{1}^{x^4}\cosh t\,dt\right)=\cosh\left(x^4\right)\frac{d}{dx}\left(x^4\right)$$
$$=4x^3\cosh\left(x^4\right).$$

■

Solution to 17.

$$\int_1^9 \frac{x-1}{\sqrt{x}}\,dx = \int_1^9 \left[\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\right]\,dx$$

$$= \int_1^9 \left[x^{1/2} - x^{-1/2}\right]\,dx$$

$$= \left[\frac{2}{3}\sqrt{x^3} - 2\sqrt{x}\right]_1^9$$

$$= \left[\frac{2}{3}(\sqrt{9})^3 - 2\sqrt{9}\right] - \left[\frac{2}{3}(\sqrt{1})^3 - 2\sqrt{1}\right]$$

$$= [18-6] - [\frac{2}{3} - 2]$$

$$= [12 - \frac{2}{3} + 2] = \frac{40}{3}.$$

■

Solution to 18.

$$\text{since} -2 \leq f(x) \leq 5 \Rightarrow \int_1^4 -2\,dx \leq \int_1^4 f(x)\,dx \leq \int_1^4 5\,dx$$

$$m \leq f(x) \leq M \Rightarrow m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

$$\Rightarrow (-2)(4-1) \leq \int_1^4 f(x)\,dx \leq 5(4-1)$$

$$\Rightarrow -6 \leq \int_1^4 f(x)\,dx \leq 15.$$

■

Solution to 19. Remember that $\int\limits_a^a f(x)\,dx = 0.$

Hence $\int\limits_3^3 \frac{\sinh^{-1} x}{2x}\,dx = 0.$ ■

Solution to 20. Let $f(x) = \frac{\sinh x}{x^4+x^2}$.

Since $f(-x) = \frac{\sinh(-x)}{(-x)^4+(-x)^2} = \frac{-\sinh x}{x^4+x^2} = -f(x)$,

then $\int\limits_{-3}^{3} \frac{\sinh x}{x^4+x^2}\,dx = 0$. ■

Solution to 21. Remember that $(\ln x)' = \frac{1}{x}$.

$$\int \frac{\sin(\ln x)}{x}\,dx = \int \sin(\ln x)\,\frac{1}{x}dx \quad \text{use the fact } \int \sin(f(x))\,f'(x)dx = -\cos(f(x)) + C.$$
$$= -\cos(\ln x) + C.$$

■

Solution to 22. Remember that $(\ln x)' = \frac{1}{x}$.

$$\int_e^{e^4} \frac{2}{x\sqrt{\ln x}}\,dx = 2\int_e^{e^4} (\ln x)^{-1/2}\,\frac{1}{x}dx \qquad \text{use the fact } \int [f(x)]^n\, f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C.$$

$$= 2\left[2(\ln x)^{1/2}\right]_e^{e^4}$$

$$= 4[\sqrt{\ln x}]_e^{e^4}$$

$$= 4[\sqrt{\ln(e^4)} - \sqrt{\ln(e)}]$$

$$= 4[\sqrt{4} - \sqrt{1}]$$

$$= 4[2-1] = 4.$$

■

Solution to 23. Remember that $(\ln x)' = \frac{1}{x}$.

$$\int x\csc^2\left(x^2\right)dx = = \frac{1}{2}\int \csc^2\left(x^2\right)2x dx \quad \text{use the fact } \int \csc^2\left(f(x)\right)f'(x)dx = -\cot\left(f(x)\right)+C.$$

$$= \frac{-\cot\left(x^2\right)}{2} + C.$$

■

Solution to 24.

$$\int_{\pi/6}^{\pi/3} \sec\theta \tan\theta \, d\theta = \int_{\pi/6}^{\pi/3} \sec\theta \tan\theta \, d\theta$$

use the fact $\int \sec(f(x)) \tan(f(x)) \, f'(x) dx = \sec(f(x)) + C.$

$$= [\sec\theta]_{\pi/6}^{\pi/3}$$

$$= [\sec(\pi/6) - \sec(\pi/3)]$$

$$= [2 - \frac{2}{\sqrt{3}}]$$

■

Solution to 25.

$$\begin{aligned}\frac{d}{dx}\left(F(x)+\operatorname{csch} x\right) == \frac{d}{dx}\left(F(x)\right)+\frac{d}{dx}\left(\operatorname{csch} x\right) \\ = f(x)+\left(-\operatorname{csch} x \coth x\right) \\ = f(x)-\operatorname{csch} x \coth x\end{aligned}$$

■

Solution to 26.

$$\int \frac{20x+4}{\sqrt[3]{5x^2+2x+1}}\,dx = = \int (5x^2+2x+1)^{-1/3}\,2(2x+2)dx$$

$$= 2\int (5x^2+2x+1)^{-1/3}\,(2x+2)dx$$

use $\int [f(x)]^n\, f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C$

$$= 2[\frac{3}{2}(5x^2+2x+1)^{2/3}] + C$$

$$= 3\sqrt[3]{(5x^2+2x+1)^2} + C.$$

■

Solution to 27.

$$\int 5^{x^2+x}(6x+3)\,dx = = \int 5^{x^2+x}\,3(2x+1)dx$$

$$= 3\int 5^{x^2+x}\,(2x+1)dx \quad \text{use } \int a^{f(x)}\, f'(x)dx = \frac{a^{f(x)}}{\ln a} + C$$

$$= 3[\frac{5^{x^2+x}}{\ln 5}] + C$$

$$= 3\frac{5^{x^2+x}}{\ln 5} + C.$$

■

Solution to 28. Note that $|3-3x| = |3(1-x)| = 3|1-x|$.

$$|1-x| = \begin{cases} 1-x, & \text{if } 1-x \geq 0; \\ -(1-x), & \text{if } 1-x < 0. \end{cases}$$

$$= \begin{cases} 1-x, & \text{if } -x \geq -1; \\ -(1-x), & \text{if } -x < -1. \end{cases}$$

$$= \begin{cases} 1-x, & \text{if } x \leq 1; \\ x-1, & \text{if } x > 1. \end{cases}$$

So on the interval $[0,1]$ we have $3|1-x| = 3(1-x)$ and on the interval $[1,2]$ we have $3|1-x| = 3(x-1)$

$$\begin{aligned}
\int_0^2 |3-3x|\,dx &= \int_0^1 |3-3x|\,dx + \int_1^2 |3-3x|\,dx \\
&= \int_0^1 3(1-x)\,dx + \int_0^1 3(x-1)\,dx \\
&= 3[x-\frac{x^2}{2}]_0^1 + 3[\frac{x^2}{2}-x]_1^2 \\
&= 3[(1-\frac{1}{2})-(0)] + 3[(\frac{4}{2}-2)-(\frac{1}{2}-1)] \\
&= 3[\frac{1}{2}-0+0+\frac{1}{2}] = 3.
\end{aligned}$$

■

Solution to 29. Since $14 = \int_{-3}^{3} g(x)\,dx = 2\int_{0}^{3} g(x)\,dx$, then $\int_{0}^{3} g(x)\,dx = 7$.

$$\int_{0}^{3} \frac{g(x)}{7}\,dx = \frac{1}{7}\int_{0}^{3} g(x)\,dx$$
$$= \frac{1}{7}7 = 1.$$

■

Solution to 30. Note that $(1+e^x)' = e^x$.

$$\int \frac{e^x}{1+e^x}\,dx = \int \frac{e^x}{1+e^x}\,dx \qquad \text{use } \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C.$$

$$= \ln(1+e^x) + C. \quad 1+e^x > 0,\ |1+e^x| = 1+e^x$$

■

Solution to 31. Note that $(1+e^{2x})' = 2e^{2x} \neq e^x$ and $e^{2x} = (e^x)^2$.

$$\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx \quad \text{use } \int \frac{f'(x)}{1+[f(x)]^2}\,dx = \tan^{-1}(f(x)) + C.$$

$$= \tan^{-1}(e^x) + C.$$

■

Solution to 32. Note that $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$.

$$\begin{aligned}\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx &= \int \sin(\sqrt{x})\,\frac{1}{\sqrt{x}}\,dx \\ &= 2\int \sin(\sqrt{x})\,\frac{1}{2\sqrt{x}}\,dx_{\text{use } \int \cos(f(x))\,f'(x)dx = \sin(f(x)) + C.} \\ &= \sin(\sqrt{x}) + C.\end{aligned}$$

Solution to 33. Let

$$u = x \qquad dv = \cos(5x)\,dx$$

$$du = x\,dx \qquad v = \frac{1}{5}\sin(5x)$$

Therefore,

$$\int x\cos(5x)\,dx = \underbrace{(x)(\frac{1}{5}\sin(5x))}_{uv} - \underbrace{\int \frac{1}{5}\sin(5x)\,dx}_{\int v\,du}$$

$$= \frac{x}{5}\cos(5x) - \int \frac{1}{5}\sin(5x)\,dx$$

$$= \frac{x}{5}\cos(5x) - \frac{1}{5}\int \sin(5x)\,dx$$

$$= \frac{x}{5}\cos(5x) - \frac{1}{5}(-\frac{1}{5}\sin(5x)) + C$$

$$= \frac{x}{5}\sin(5x) + \frac{1}{25}\cos(5x) + C.$$

■

Solution to 34.

Alternate signs	u and its derivatives	dv and its antiderivatives
$+$	$3x^2$	e^{5x}
$-$	$6x$	$\frac{1}{5}e^{5x}$
$+$	6	$\frac{1}{25}e^{5x}$
$-$	0	$\frac{1}{125}e^{5x}$

Differentiate until you get 0.

Hence $\int 3x^2 e^{5x}\,dx = \frac{3}{5}x^2e^{5x} - \frac{6}{25}xe^{5x} + \frac{6}{125}e^{5x} + C.$ ■

Solution to 35. Let

$$u = \cos x \qquad dv = e^x\,dx$$
$$du = -\sin x\,dx \qquad v = e^x$$

Therefore,

$$\int e^x \cos x\, dx = e^x \cos x - \int e^x(-\sin x)\, dx$$ Use $\int u\, dv = uv - \int v\, du$.

$$= e^x \cos x + \int e^x \sin x\, dx$$ Int. by part again $u = \sin x \quad dv = e^x$

$du = \cos x\, dx \quad v = e^x$.

$$= e^x \cos x + \left[e^x \sin x - \int e^x \cos x\, dx\right]$$ Use $\int u\, dv = uv - \int v\, du$.

$$\int e^x \cos x\, dx = e^x \cos x + e^x \sin x - \int e^x \cos x\, dx$$ We are trying to find $\int e^x \cos x\, dx$.

$$\int e^x \cos x\, dx + \int e^x \cos x\, dx = e^x \cos x + e^x \sin x$$ Add the two integral.

$$2\int e^x \cos x\, dx = e^x \cos x + e^x \sin x$$ Divide by 2 and add C to the right side

$$\int e^x \cos x\, dx = \frac{e^x \sin x + e^x \cos x}{2} + C.$$

■

Solution to 36. Let

$$u = \ln(2x-1) \qquad\qquad dv = dx$$

$$du = \frac{2}{2x-1}\,dx \qquad\qquad v = x$$

Therefore,

$$\int \ln(2x-1)\,dx = x\ln(2x-1) - \int x\frac{2}{2x-1}\,dx \qquad \text{Use } \int u\,dv = uv - \int v\,du\ .$$

$$= x\ln(2x-1) - \int \frac{2x}{2x-1}\,dx \qquad \text{use } \frac{2x-1+1}{2x-1}$$

$$= x\ln(2x-1) - \int \left[\frac{2x-1}{2x-1} + \frac{1}{2x-1}\right]dx \qquad \text{Use } \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\ .$$

$$= x\ln(2x-1) - \int \left[1 + \frac{1}{2x-1}\right]dx$$

$$= x\ln(2x-1) - \int 1\,dx - \int \frac{1}{2x-1}\,dx$$

$$= x\ln(2x-1) - x - \frac{1}{2}\int \frac{2}{2x-1}\,dx \qquad \int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C.$$

$$= x\ln(2x-1) - x - \frac{1}{2}\ln(2x-1) + C$$

$$= (x - \frac{1}{2})\ln(2x-1) - x + C$$

$$= \frac{2x-1}{2}\ln(2x-1) - x + C.$$

■